(Pages : 4) J – 3069

Reg. No. : .....

Name : .....

**Fourth Semester B.Sc. Degree Examination, June 2020**

**First Degree Programme under CBCSS**

**Mathematics**

**Complementary Course for Physics**

**MM. 1431.1 MATHEMATICS IV (COMPLEX ANALYSIS, FOURIER SERIES AND FOURIER TRANSFORMS)**

**(2014-2017 Admission)**

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Reduce $\frac{5i}{(1-i)(2-i)(3-i)}$ into a real number.

2. Find the value of $(3,1)(3,-2)$.

3. Find the principal argument $Arg\, z$, where $z = \left(\sqrt{3} - i\right)^6$.

4. Find the real an imaginary parts of the function $f(z) = e^z$.

5. Write the Maclaurin series expansion of $\frac{1}{1+z}$.

P.T.O.

6. Find the singularities of the function $f(z) = \frac{1}{z + z^2}$.

7. Find the residue at $z = 0$ of the function $\frac{5z - 2}{z(z-1)}$.

8. Evaluate $\int_C \frac{dz}{z^2 + 4z + 4}$ where $C: |z| = 1.5$.

9. Write the standard form of Fourier series expansion of a function of period $2L$.

10. Write the Euler formula for Fourier Co-efficient of a function of period $2\pi$.

## SECTION – II

Answer any **eight** question from among the questions 11 to 22. They carry **2** marks each.

11. Sketch the set of points determined by the condition $|z - 1| = 4$.

12. Show that an analytic function with constant imaginary part is a constant.

13. Write the function $f(z) = z - \frac{1}{z}$, $z \neq 0$ in the form $f(z) = U(r, \theta) + iV(r, \theta)$.

14. Find the principal value of $i^i$.

15. Write the real and imaginary parts of $\cos z$.

16. Find the value of $\int_C \bar{z}\, dz$ where $C$ is the upper half of the circle $|z| = 2$ oriented clockwise.

17. Evaluate $\int_C \frac{z^2\, dz}{z - 3}$ where $C: |z| = 5$.

18. Locate the singularity of $\frac{\sinh z}{z^4}$ and determine its nature.

19. Find the residue of $\frac{1-\cosh z}{z^3}$ at $z=0$.

20. Find the Fourier Sine Transform of the function $f(x)=1, \text{if } 0<x<c$
$=0, \text{if } x>c$.

21. Find the Fourier Cosine Transform of the function $f(x)=e^{-x}, x>0$.

22. Show that $F_c[f(ax)]=\frac{1}{a}F_c\left(\frac{w}{a}\right), a\neq 0$.

## SECTION – III

Answer any **six** questions from among the questions 23 to 31. They carry **4** marks each.

23. Show that the function $f(z)=z^2$ is analytic everywhere and find its derivative.

24. Prove that $f'(z)$ does not exist for $f(z)=e^x e^{-iy}$.

25. Prove that $f(z)=u+iv$ is analytic in a domain if and only if $v$ is a harmonic conjugate of $u$.

26. Evaluate $\int_0^{1+i}(x^2-iy)dz$ along the path $y=x^2$.

27. Evaluate $\int_C \frac{(3x^2+z)dz}{(z^2-1)}$ where $C:|z-1|=1$.

28. Determine the poles of the function $f(z)=\frac{z^2}{(z-1)(z-2)^2}$ and the residue at each pole.

29. Find the Fourier series expansion of the function $f(x)=x^3, -\pi<x<\pi$.

30. Find the half range cosine series of the function $f(x) = \pi - x, 0 < x < \pi$.

31. Find the Fourier transform of the function $f(x) = xe^{-x}$, if $x > 0$
$= 0$, if $x < 0$.

## SECTION – IV

Answer any **two** questions from among the questions 32 to 35. They carry **15** marks each.

32. (a) Obtain the harmonic conjugate of $U(x, y) = 2x - x^3 + 3xy^2$.

(b) Let $f(z) = U + iV$ be an analytic function. Find $f(z)$ if $U + V = \frac{x}{x^2 + y^2}$.

33. Expand $\frac{1}{(z-1)(z-2)}$ in the regions.

(a) $|z| < 1$

(b) $1 < |z| < 2$

(c) $|z| > 2$.

34. Find a Fourier series to represent $e^{-x}$ from $x = -l$ to $l$.

35. Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| \leq 1$
$= 0$, otherwise. Hence prove that $f(x) = \frac{2}{\pi}\int_0^\infty \frac{(1-\cos w)\cos w\, x\, dw}{w^2}$. Also deduce that $\int_0^\infty \frac{(1-\cos w)\, dw}{w^2} = \frac{\pi}{2}$.

---